Homework 1

Please complete following problems.

1. Prove that the inverse element defined in (VS4) is unique.
2. [FIS P12 1.] Label the following statements as true or false.
   (a) Every vector space contains a zero vector.
   (b) A vector space may have more than one zero vector.
   (c) In any vector space, $ax = bx$ implies that $a = b$.
   (d) In any vector space, $ax = ay$ implies that $x = y$.
   (e) A vector in F^n may be regarded as a matrix in $M_{n\times 1}(F)$.
   (f) An $m \times n$ matrix has m columns and n rows.
   (g) In $P(F)$, only polynomials of the same degree may be added.
   (h) If f and g are polynomials of degree n, then $f + g$ is a polynomial of degree n.
   (i) If f is polynomial of degree n and c is a nonzero scalar, then cf is a polynomial of degree n.
   (j) A nonzero scalar of F may be considered to be a polynomial in $P(F)$ having degree zero.
   (k) Two functions in $\mathscr{F}(S,F)$ are equal if and only if they have the same value at each element of S.
3. [FIS P14 7.] Let $S = \{0,1\}$ and $F = R$. In $\mathscr{F}(S,R)$, show that $f = g$ and $f + g = h$ where $f(t) = 2t+1$, $g(t) = 1+4t-2t^2$, and $h(t) = 5^t + 1$.
4. [FIS P14 8.] In any vector space V, show that $(a+b)(x+y) = ax+ay+bx+by$ for and $x, y \in V$ and any $a, b \in F$.
5. [FIS P15 12.] A real-valued function f defined on the real line is called an even function if $f(-t) = f(t)$ for each real number t. Prove the set of even functions defined on the real line with the operations of addition and scalar multiplication defined in Example 3 is a vector space.
6. [FIS P16 19.] Let $V = \{(a_1, a_2) : a_1, a_2 \in R\}$. Define addition of elements of V coordinatewise, and for (a_1, a_2) in V and $c \in R$, define

$$c(a_1, a_2) = \begin{cases} (0,0) & \text{if } c = 0 \\ (ca_1, \dfrac{a_2}{c}) & \text{if } c \neq 0. \end{cases}$$

   Is V a vector space over R with these operations? Justify your answer.