Homework 5

Please complete following problems.

1. Recall from Lecture 7 page 6.

   We rigorously justify the validity of the solution of the form

   $$b_0 q(x)^{-1/4} e^{\frac{1}{\varepsilon}\int^x \sqrt{q(s)}ds} (1 + O(\varepsilon)).$$

   The other part can be obtained by working with $\tilde{\Phi} := w_2/w_1$. We also note that in this case we shall integrate in the backward direction when defining the contraction mapping to gain smallness from the exponential part in the case $q > 0$. Such solution is independent from the foregoing one because at $x = 0$ $|w_2(0)/w_1(0)| < \infty$ while the ratio of the foregoing one is infinite.

   Define $\Psi(s) := w_2(s)/w_1(s)$ and compute the ODE satisfied by $\Psi(s)$. Impose the initial condition $\Psi(L/\varepsilon) = 0$ and write down the corresponding integral equation which leads to the definition of an operator on

   $$X := \{\Psi : \Psi \in C^b([0, L/\varepsilon], \mathbb{C}),\ \Psi(L/\varepsilon) = 0\}.$$

   (5 points).

   Rigorously justify the validity of the other solution of the form

   $$a_0 q(x)^{-1/4} e^{\frac{1}{\varepsilon}\int^x -\sqrt{q(s)}ds}\big(1 + O(\varepsilon)\big).$$

   by working out the sections 4.1, 4.2, and 4.3 from Lecture 7 for Ψ. (5 points for each section.)

2. Excise 4.1 (b) (10 points)

3. Excise 4.16 (c) (10 points)