Please complete following problems.

1. Read example 3 on page 9 and section 1.4.2.
2. Excise 1.4. (10 points)
3. Excise 1.6. (c) and (h) (10 points)
4. Excise 1.16 (10 points) We shall follow Examples 2 and 3 on page 16-18 to resolve this problem.

   (a). For x close to zero, we find
$$\frac{1}{\sqrt{1-x\sin^2(s)}} = 1+\frac{1}{2}\sin^2(s)x+\mathcal{O}(x^2), \quad \text{uniformly for } s\in[0,\frac{\pi}{2}],$$
   whence
$$\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-x\sin^2(s)}}ds = \frac{\pi}{2}+\frac{\pi}{8}x+\mathcal{O}(x^2).$$
   Thus for x close to zero, $K \sim \frac{\pi}{2}+\frac{\pi}{8}x$.

   (b) and (c). For x close to one, the integrand $\frac{1}{\sqrt{1-x\sin^2(s)}}$ is large when s is close to $\frac{\pi}{2}$. Set $\epsilon := 1-x$ and $u := \frac{\pi}{2}-s$. The integral amounts to
$$\int_0^{\frac{\pi}{2}} \frac{du}{\sqrt{\sin^2(u)+\epsilon\cos^2(u)}}.$$
   Note that
$$\frac{1}{\sqrt{\sin^2(u)+\epsilon\cos^2(u)}} = \frac{1}{\sin(u)}\frac{1}{\sqrt{1+\epsilon\cot^2(u)}} \sim \frac{1}{\sin(u)}\left(1-\frac{\epsilon}{2\tan^2(u)}+\frac{3\epsilon^2}{8\tan^4(u)}+\cdots\right).$$
   For this to be well ordered near $u=0$ we require that $\epsilon \ll \tan^2(u)$ or $\epsilon \ll u^2$. So we write
$$K(\epsilon) = \int_0^{\delta} \frac{du}{\sqrt{\sin^2(u)+\epsilon\cos^2(u)}} + \int_\delta^{\frac{\pi}{2}} \frac{du}{\sqrt{\sin^2(u)+\epsilon\cos^2(u)}} \quad \text{with } \sqrt{\epsilon} \ll \delta.$$
   The latter integral asymptotically expands as
$$\begin{aligned}
&\int_\delta^{\frac{\pi}{2}} \frac{du}{\sqrt{\sin^2(u)+\epsilon\cos^2(u)}} \\
&\sim \int_\delta^{\frac{\pi}{2}} \frac{1}{\sin(u)} - \frac{1}{2\sin(u)\tan^2(u)}\epsilon + \cdots du \\
&= \ln\left(\tan\left(\frac{u}{2}\right)\right)\Big|_\delta^{\frac{\pi}{2}} + \left(\frac{\ln\left(\tan\left(\frac{u}{2}\right)\right)}{4} + \frac{1}{16\tan\left(\frac{u}{2}\right)^2} - \frac{\tan\left(\frac{u}{2}\right)^2}{16}\right)\Big|_\delta^{\frac{\pi}{2}}\epsilon + \cdots \\
&= -\ln\left(\tan\left(\frac{\delta}{2}\right)\right) + \left(\frac{\tan\left(\frac{\delta}{2}\right)^2}{16} - \frac{1}{16\tan\left(\frac{\delta}{2}\right)^2} - \frac{\ln\left(\tan\left(\frac{\delta}{2}\right)\right)}{4}\right)\epsilon + \cdots \\
&\sim -\ln(\delta) + \ln(2) - \frac{1}{4\delta^2}\epsilon + \cdots
\end{aligned}$$

Setting $\eta = \delta/\sqrt{\epsilon}$ and $r = u/\sqrt{\epsilon}$, the foregoing integral becomes

$$
\begin{aligned}
&\int_0^\delta \frac{du}{\sqrt{\sin^2(u)+\epsilon\cos^2(u)}} = \int_0^\eta \frac{\sqrt{\epsilon}dr}{\sqrt{\sin^2(\sqrt{\epsilon}r)+\epsilon\cos^2(\sqrt{\epsilon}r))}} \\
&\sim \int_0^\eta \frac{\sqrt{\epsilon}dr}{\sqrt{(1+r^2)\epsilon-(r^2+\frac{1}{3}r^4)\epsilon^2+(\frac{1}{3}r^4+\frac{2}{45}r^6)\epsilon^3+\cdots}} \\
&= \int_0^\eta \frac{dr}{\sqrt{(1+r^2)-(r^2+\frac{1}{3}r^4)\epsilon+(\frac{1}{3}r^4+\frac{2}{45}r^6)\epsilon^2+\cdots}} \\
&\sim \int_0^\eta \frac{1}{\sqrt{1+r^2}}+\frac{r^2(r^2+3)}{6(r^2+1)^{3/2}}\epsilon+\cdots dr \\
&= \ln\left(r+\sqrt{1+r^2}\right)\Big|_0^\eta + \left(\frac{1}{4}\ln\left(r+\sqrt{1+r^2}\right)+\frac{r\left(r^2-3\right)}{12\sqrt{r^2+1}}\right)\epsilon\Big|_0^\eta+\cdots \\
&= \ln\left(\eta+\sqrt{1+\eta^2}\right)+\left(\frac{1}{4}\ln(\eta+\sqrt{1+\eta^2})+\frac{\eta\left(\eta^2-3\right)}{12\sqrt{\eta^2+1}}\right)\epsilon+\cdots \\
&= (1+\frac{1}{4}\epsilon)\ln\left(\eta+\sqrt{1+\eta^2}\right)+\frac{\eta\left(\eta^2-3\right)}{12\sqrt{\eta^2+1}}\epsilon+\cdots \\
&\sim (1+\frac{1}{4}\epsilon)\left(\ln(2\eta)+\frac{1}{2\eta^2}\right)+\frac{1}{12}\eta^2\epsilon+\cdots \\
&= (1+\frac{1}{4}\epsilon)\left(\ln(2)+\ln(\delta)-\frac{1}{2}\ln(\epsilon)+\frac{\epsilon}{2\delta^2}\right)+\frac{1}{12}\delta^2\cdots
\end{aligned}
$$

Summing up yields the conclusions in (b) and (c).

5. Excise 1.18. (e) and (k) (10 points)